RESEARCH INTERNSHIP REPORT

Optimizing Urban Redevelopment: An Operational Approach to Land Use and Transportation

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ABSTRACT

Urban redevelopment is essential for enhancing the quality of life, promoting sustainability, and revitalizing communities. This research project focuses on optimizing land use allocation and transportation network design within urban redevelopment sites, particularly areas facing abandonment. By integrating land use planning with transportation optimization, the study aims to support sustainable modes of transport, including cycling and walking, thereby reducing car dependence and fostering healthier urban environments.

The project introduces a comprehensive linear optimization model that strategically allocates land use and enhances bike lane networks while considering the interdependent nature of these elements. This model was applied to a case study of an abandoned mall, demonstrating how optimized redevelopment plans can effectively utilize resources to maximize community benefits. The findings provide urban planners with data-driven insights to compare the outcomes of various redevelopment strategies, offering a valuable contribution to the field.

Preliminary results indicate that this model presents an innovative approach to urban redevelopment, particularly in analyzing the interactions between broader transportation networks and internal site infrastructure. While the results are promising, they underscore the need for more accurate data, particularly regarding land-use attractiveness coefficients. Future research should focus on optimizing internal routes and incorporating interactions between different land uses to further enhance the model's utility. Overall, this study lays the groundwork for more resilient and livable urban environments through informed and strategic redevelopment practices.

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1 INTRODUCTION

Background and Motivation

Urban redevelopment is a critical aspect of city planning, aiming to improve the quality of life for residents, promote sustainable development, and revitalize communities. As cities grow and evolve, the need for efficient land use allocation and sustainable transportation systems becomes increasingly important. Traditional car-centric urban designs are increasingly being challenged by the need for greener and more efficient transportation solutions that reduce congestion, lower emissions, and improve public health.

During this internship I had the opportunity to contribute to this field by working on a project that combined urban planning with optimization techniques. This project was motivated by the pressing need to rethink how urban spaces are utilized and how transportation networks can be designed to support alternative, sustainable modes of transport such as cycling and walking. Importantly, this work is not intended to replace architectural expertise but to complement it, providing urban planners with new perspectives on community needs and potential solutions they might not have considered.

The focus of this project was on sites within cities that are facing abandonment, such as bankrupt malls or defunct state services with extensive parking areas around them. These sites represent significant opportunities for redevelopment to better serve the community's needs. The long-term objective is to equip urban planners with numerical data that can be used to compare the attractiveness and effectiveness of various redevelopment plans for the community. By providing quantifiable insights, the project aims to enhance the decision-making process, ensuring that redevelopment efforts are both innovative and responsive to the evolving demands of urban populations.

Objectives of the Study

The primary objective of this research was to optimize land use allocation within a redevelopment site that had undergone significant changes. Effective land use planning is essential for maximizing the functional and economic potential of urban areas while minimizing environmental impact. This aspect of the project sought to ensure that the redeveloped site would meet the diverse needs of the community, including residential, commercial, and recreational spaces, in an efficient and sustainable manner.

Simultaneously, the project aimed to design and optimize the transportation network around the site, with a particular focus on bike lanes. The goal was to promote alternative means of transport, reducing reliance on cars and fostering a more sustainable urban environment. By prioritizing biking, the project addressed the growing demand for healthier, safer, and more environmentally friendly transportation options.

Addressing these two objectives concurrently was crucial, as it allowed for the examination of the interconnections between land use and transportation. Understanding how these elements interact can lead to more integrated and effective urban planning strategies. The research aimed to demonstrate how optimized land use allocation and a well-designed transportation network could work together to enhance the overall functionality and sustainability of the urban area. This integrated approach is vital for ensuring that urban redevelopment projects not only meet immediate needs but also contribute to long-term urban resilience and livability.

Project Contribution

The project makes contributions to the field of urban redevelopment by proposing a comprehensive linear optimization model that integrates land use allocation with transportation network design. This model aims to enhance both connectivity and livability within urban areas by strategically planning land use and improving bike lane networks, while also simulating the choices of inhabitants. By doing so, the project addresses the dual objectives of sustainable land use and efficient transportation simultaneously, recognizing the interdependent nature of these elements.

In practical terms, the project applied this model to a case study involving an ancient mall within a city facing abandonment. The model's application presented some strategic land use allocations and bike lanes improvements that could significantly improve the attractiveness and functionality of the redevelopment site. The results showed optimized plans that effectively utilized the city's budget to maximize the positive impact on the community, making the area more accessible, sustainable, and vibrant.

The project's findings provide urban planners with new insights and numerical data to compare the potential outcomes of various redevelopment strategies. This contribution is particularly important as it offers a data-driven approach to urban planning, helping to quantify the benefits of different plans and ensuring that redevelopment efforts are both effective and responsive to the needs of the community. Overall, the project paves the way for more informed and strategic urban redevelopment practices, ultimately contributing to the creation of more resilient and livable urban environments.

2
LITERATURE REVIEW

Urban redevelopment involves optimizing the allocation of land use and the design of transportation networks to create more sustainable and livable environments. Various studies have contributed to understanding and improving these processes, offering insights into connectivity, compactness, compatibility, and the integration of green spaces.

• Connectivity and Service Utilities

To begin with, [Li et al, 2019] propose a method to estimate the quality of an urban network based on the connectivity between service providers and customers, which depends on distance. When working within a limited budget, the optimal solution often involves considering customers as clusters to maximize connectivity. This approach is critical for urban planners seeking to enhance service accessibility and overall network efficiency.

• Land Use Optimization

Building on the concept of connectivity, [Rahman and Szabó, 2021] emphasize three primary objectives in urban land use optimization: maximizing spatial compactness, land use compatibility, and land use suitability. Spatial compactness reduces emissions, promotes walking and biking, and conserves rural areas. Compact city development leads to higher social interaction, personal satisfaction, and perceived health benefits. Land use compatibility ensures that adjacent land uses do not create adverse effects, fostering economic vitality, community soundness, and social interaction. These objectives align with sustainable urban development goals, making them essential considerations in redevelopment projects.

Moreover, [Ligmann-Zielinska et al, 2005] define a mathematical model for optimizing sustainable land use allocation under area and budget constraints. The model's objectives include minimizing open space development, redevelopment, incompatibility of adjacent land uses, and distance to already developed areas. These strategies balance economic, environmental, and social equity considerations, providing a comprehensive framework for urban planners. This model is particularly relevant for redeveloping abandoned urban sites, ensuring efficient and sustainable land use.

• Pedestrian Infrastructure and Behavior

In parallel, pedestrian infrastructure plays a crucial role in urban redevelopment. [Nabipour et al, 2022] analyze pedestrian commuting behavior in intermediate-sized cities, identifying three key factors: network design, built environment, and safety features. Network design includes street connectivity and sidewalk continuity, while the built environment encompasses land use mix and access to public transit. Safety features involve vehicle speed control and intersection traffic management. The study utilizes a multinomial logit (MNL) model, where the likelihood result is proportional to the exponential of the utility, to simulate pedestrian choices, providing valuable insights for designing pedestrian-friendly urban areas. Understanding these factors is essential for creating inclusive and accessible urban environments.

- ## Bicycle Infrastructure Planning

In addition to pedestrian infrastructure, [Liu et al, 2019] present a model for planning bike lanes using bike trajectories, focusing on coverage and continuity. Bikers prefer continuous bike lanes over those that switch to car roads, as changes can be dangerous. The model optimizes bike lane implementation within a city's road network, considering origin-destination routes. This approach enhances biker safety and encourages cycling as a sustainable transport mode.

An extension of this work by [Liu et al, 2022] incorporates the impact of new bike lanes and bikers on car traffic. The study aims to maximize the number of bikers by strategically placing bike lanes and introduces a method to linearize the multinomial logit (MNL) model for use with mixed-integer linear programming (MILP) methods. This integration of traffic dynamics provides a more holistic view of urban transportation planning.

- ## Urban Green Spaces

Finally, the integration of green spaces into urban areas is a critical component of sustainable redevelopment. [Salgado et al, 2022] explore the exposure to parks through the lens of urban mobility. They calculate park exposure based on the number of parks surrounding daily activities and the demand for parks based on the number of activities around them. The study shows that a better distribution of parks, even with smaller individual areas, provides more equitable access to green spaces. Urban green spaces promote mental and physical health, reduce morbidity and mortality, and support social cohesion. This research highlights the importance of integrating green spaces into urban redevelopment plans and aligns with the broader goals of enhancing urban livability and sustainability.

- ## Conclusion

In conclusion, the reviewed literature underscores the importance of optimizing land use and transportation networks to create sustainable urban environments. By considering connectivity, compactness, compatibility, and the integration of green spaces, urban planners can develop strategies that enhance the functionality and livability of urban areas. These studies provided valuable frameworks that were useful to this redevelopment project, offering specific perspectives on each objective of the project.

3
MODELS

In our project, we aim to tackle an optimization problem that local governments face when developing mixed-use zones and infill redevelopment areas. Our goal is to enhance social welfare by optimizing both the land-use proportions of the development site and its connectivity to the surrounding neighborhoods. By addressing these two aspects, we ensure that the site not only meets developmental goals but also integrates seamlessly with existing community structures, thereby promoting accessibility and utility.

The optimization approach involves two perspectives: a holistic view of the site as a single entity, focusing on the total area to be optimized, and a granular view that examines the site as comprising multiple zones, each with its distinct area to be optimized.

3.1 Holistic Model

To define our model, let $\mathcal{I}$ denote the set of nearby neighborhoods, with each neighborhood $i \in \mathcal{I}$ having a population I_i. We define $\mathcal{A}_i$ as the set of the n^{th} shortest routes that connect neighborhood i to the development site, ideally through improved bike lanes. If a resident from neighborhood i accesses the site via a specific route $C \in \mathcal{A}_i$, they gain a utility $u_{i,C}$. If they opt not to use any route to the site, they have two other options: driving with a utility u_i^D and an outside option (typically staying home or going somewhere else) with a utility u_i^S. Using the multinomial logit (MNL) model, the probability that a person from i will choose to bike to the site is given by:

$$p_i^B = \frac{\sum_{C \subset \mathcal{A}_i} u_{i,C}^B}{u_i^S + u_i^D + \sum_{C \subset \mathcal{A}_i} u_{i,C}^B}$$

and the probability they use route C is:

$$p_{i,C}^B = \frac{u_{i,C}^B}{u_i^S + u_i^D + \sum_{C \subset \mathcal{A}_i} u_{i,C}^B}$$

The total number of people from neighborhood i biking to visit the site is then given by $I_i p_i^B$.

When it comes to site development, the process is typically undertaken by a private firm, with the government setting specific requirements. These requirements are expressed through decision variables $\mathbf{x}_k$ for $k \in \mathcal{K}$, where $\mathcal{K}$ represents the types of development (e.g., retail, residential, green space, parking). $\mathbf{x}_k$ indicates the minimum (or maximum) area required for each type, subject to the constraint $\sum_k \mathbf{x}_k \leq M^A$, where M^A is the total site area.

Thus, $\mathbf{w}$ being the variable representing the installation or improvement of each road segment, the government's decision variables can be encapsulated in a tuple $(\mathbf{w}, \mathbf{x})$, which aims to maximize social welfare.

Given the government's decisions $(\mathbf{w}, \mathbf{x})$, the private developer solves an inner optimization problem to maximize revenue or profit under a maximum cost constraint. The developer's decision variables are $\tilde{\mathbf{x}}_k$ for $k \in \mathcal{K}$, which is the actual area allocated for each development type, subject to the constraints $\tilde{\mathbf{x}}_k \geq \mathbf{x}_k$ (or $\tilde{\mathbf{x}}_k \leq \mathbf{x}_k$ if $\mathbf{x}_k$ is the maximum area).

In this project, we simplify the problem to recommend an optimal land-use allocation within the site and an optimized bike lane network around it. Thus, $\mathbf{x}_k$ will represent the optimal area rather than a minimum or maximum constraint. Consequently, we will here only consider the government's problem, assuming the government has the final decision on the site distribution (which is not the case in practice).

3.1.1 • Detailed Model

We define the following notations:

$$
\begin{aligned}
\mathcal{I} &: \text{ set of nearby neighborhoods} \\
I_i &: \text{ population of neighborhood } i \in \mathcal{I} \\
\mathcal{A}_i &: \text{ the set of the } n^{th} \text{ shortest routes connecting } i \text{ to the site} \\
C \subset \mathcal{A}_i &: \text{ a route, made of segments, connecting } i \text{ to the site} \\
\mathcal{S} &: \text{ the set of all road segments} \\
\mathcal{M} = \{B, D, S\} &: \text{ the set of transportation modes} \\
\mathbf{w} &: \text{ government decision variable in } \{0,1,2\}^{|\mathcal{S}|} \text{ indicating bike lane development} \\
\mathcal{K} &: \text{ set of all development types (e.g., residential, parking, commercial, green space)} \\
\mathbf{x} &: \text{ government decision variable in } \mathbb{R}_+^{|\mathcal{K}|} \text{ indicating on site distribution} \\
P_0 &: \text{ the decided number of parking spots on the site, proportional to } \mathbf{x}_{\text{Parking}} \\
M^A &: \text{ total area of the site} \\
M^B &: \text{ total bike lane budget}
\end{aligned}
$$

Transportation Modes and Utilities

We consider three commuting choices for residents: biking to the site (denoted by W), driving to the site (denoted by D), and the outside option (denoted by S). For each mode, we define:

$$\begin{aligned} u^B_{i,C} &: \text{ utility of walking from } i \text{ to the site via path } C \\ u^D_i &: \text{ utility of driving from } i \text{ to the site} \\ u^S_i &: \text{ utility of not visiting the site} \end{aligned}$$

For the general population outside the nearby neighborhoods ($i = 0$), only driving or not visiting are options.

For the on-site population living in the potential residential units ($i = 1$), only biking or not visiting are options. Let $k = 1$ denote residential development, the on-site population is considered proportional, with a factor ρ_1 representing the inhabitants density within a residential area, to the on-site residential area given by $\mathbf{x}_1$.

The utilities $u^W_{i,C}$ and u^D_i depend on various factors, including the number of bike lanes, their connectivity, and the attractiveness of the site. Specifically, let $C = \{s^1_C, ..., s^{n_C}_C\}$ the ordered decomposition of the route with all its segments, $\|C\|$ its total length and $\|\tilde{C}^D_i\|$ the total length of the shortest driving path from i to the site,

$$\begin{aligned} u^B_{i,C} &= \sum_{k\in\mathcal{K}} \alpha_{i,k}\, \mathbf{x}_k - \lambda^B_1 \|C\| + \lambda^B_2 \sum_{l=1}^{n_C} \mathbf{w}_{s^l_C} \|s^l_C\| + \lambda^B_3 \left(\sum_{l=1}^{n_C-1} \mathbb{1}(\mathbf{w}_{s^l_C}\, \mathbf{w}_{s^{l+1}_C} > 0) \right) \left(\frac{\|C\|}{n_C - 1} \right) \\ u^D_i &= \sum_{k\in\mathcal{K}} \alpha_{i,k}\, \mathbf{x}_k - \lambda^D_1 \|\tilde{C}^D_i\| - \lambda^D_2 f_P(\mathbf{x}) \\ u^S_i &= \beta^S = 0 \end{aligned}$$

Each term in these formulas represents specific factors contributing to the overall utility. The first utility $u^B_{i,C}$ is composed of four terms. The first term, $\sum_{k\in\mathcal{K}} \alpha_{i,k} x_k$, represents the site's attractiveness, which depends on the area $\mathbf{x}_k$ and attractiveness $\alpha_{i,k}$ of each land-use type k in the development. The second term, $\lambda^B_1 \|C\|$, represents the time needed to reach the site by bike, which is proportional to the route's total length $\|C\|$, adjusted by λ^B_1, proportional to the biking average speed. The third term, $\lambda^B_2 \sum_{l=1}^{n_C} w_{s^l_C} \|s^l_C\|$, provides a coverage bonus if the route includes bike lanes. Here, $w_{s^l_C}$ indicates whether segment s^l_C is a bike lane and its quality (no bike lane $\to 0$, normal bike lane $\to 1$, upgraded bike lane $\to 2$), and $\|s^l_C\|$ is its length. This term ensures that a fully covered route gives a bonus of $2\lambda^B_2$ times the total length. The fourth term, $\lambda^B_3 \left(\sum_{l=1}^{n_C-1} \mathbb{1}(\mathbf{w}_{s^l_C}\, \mathbf{w}_{s^{l+1}_C} > 0) \right) \left(\frac{\|C\|}{n_C-1} \right)$, adds a continuity bonus if bike lane segments are continuous. It rewards the uninterrupted segments of bike lanes, contributing to a smoother biking experience, and ensures that a fully covered route gives a bonus of λ^B_3 times the total length.

Having this formula, we see that with a route fully covered by an upgraded ($\mathbf{w} = 2$) bike lane, the time malus will be partly counterbalanced by the coverage and continuity bonuses having $u^B_{i,C} = \sum_{k\in\mathcal{K}} \alpha_{i,k}\, \mathbf{x}_k - (\lambda^B_1 - 2\lambda^B_2 - \lambda^D_3)\|C\|$. We then would take λ^B_2 and λ^B_2 as percentages of λ^B_1.

The second utility, u_i^D, consists of two terms. The first term, $\sum_{k\in\mathcal{K}} \alpha_{i,k} x_k$ remains the site's attractiveness, similar to the biking utility. The second term $\lambda_1^D \|\tilde{C}_i^D\|$ represents the time to drive to the site, and the third term $\lambda_2^D f_P(\mathbf{x})$ accounts for the potential inconvenience caused by a lack of parking spots, with f_P a function defined below that simulates the number of lacking parking spots.

Finally, the third utility, u_i^S, is a constant term denoted as $\beta^S = 0$. This utility is a constant since not visiting the site does not depend on the specific neighborhood i.

Estimating the Lack of Parking Spots f_P

We can consider that $I_i\ p_i^D(t) \frac{\alpha_{i,k}(t)\,\mathbf{x}_k}{\sum_{k'} \alpha_{i,k'}(t)\,\mathbf{x}_{k'}}$ corresponds to the number of people from neighborhood i wishing to arrive at the development site at time t because they are interested in land-use k.

Considering T_k as the average time spent by an individual at land-use k, we then have:

- $P(t) = \sum_{i,k} \sum_{t'=t-T_k}^{t-1} I_i\ p_i^D(t) \frac{\alpha_{i,k}(t)\,\mathbf{x}_k}{\sum_{k'} \alpha_{i,k'}(t)\,\mathbf{x}_{k'}}$ represents the number of cars in the parking lot (potentially some of them waiting).
- $P^+(t) = \sum_{i,k} I_i\ p_i^D(t) \frac{\alpha_{i,k}(t)\,\mathbf{x}_k}{\sum_{k'} \alpha_{i,k'}(t)\,\mathbf{x}_{k'}} = \sum_i I_i\ p_i^D(t)$ represents the number of arriving cars in the parking lot.
- $P^-(t) = \sum_{i,k} I_i\ p_i^D(t - T_k) \frac{\alpha_{i,k}(t-T_k)\,\mathbf{x}_k}{\sum_{k'} \alpha_{i,k'}(t-T_k)\,\mathbf{x}_{k'}}$ represents the number of leaving cars in the parking lot.

Such that $P(t+1) = P(t) - P^-(t) + P^+(t)$. If some of the cars were waiting at time t, we still have:

$$P(t+1) = P_0 - P^-(t) + (P(t) - P_0) + P^+(t)$$

with P_0 being the number of parking spots.

Then, the waiting time for someone arriving at time t can be written as:

$$\Delta_t = \min\{\Delta \geq 0 \quad \text{s.t.} \sum_{t'=t+1}^{t+\Delta} P^-(t') \geq (P(t) - P_0)\mathbb{1}_{P(t)\geq P_0}\}$$

With stationary hypothesis (α and p independent of t), we can now consider that

$$P(t) = P = \sum_{i,k} I_i\ p_i^D \frac{\alpha_{i,k}\,\mathbf{x}_k}{\sum_{k'} \alpha_{i,k'}\,\mathbf{x}_{k'}}\ T_k$$

and the waiting time becomes:

$$\Delta_t = \mathbb{1}_{P\geq P_0} \left\lceil \frac{P - P_0}{P^-} \right\rceil = \mathbb{1}_{P\geq P_0} \left\lceil \frac{P - P_0}{\sum_i I_i\ p_i^D} \right\rceil \approx \mathbb{1}_{P\geq P_0} \left(\frac{P - P_0}{\sum_i I_i\ p_i^D} \right)$$

Wishing that f_P be linear (with also the possibility to include indicator variables), we will choose to simplify the expression of P using $\hat{p}_i^D$ and $\hat{\mathbf{x}}_k$, estimators of p_i^D and $\mathbf{x}_k$. The choice of the estimators precision remains open, but even very simple estimators such as $\hat{p}_i^D = 0.5$ and $\hat{\mathbf{x}}_k = \frac{M^A}{|\mathcal{K}|}$ are not without merit.

Using these estimators, we can explicitely express f_P:

$$f_P(\mathbf{x}) = (P - P_0)\mathbb{1}_{P \geq P_0} \approx (\hat{P}(\mathbf{x}) - P_0)\mathbb{1}_{\hat{P}(\mathbf{x}) \geq P_0}$$

with $\hat{P}(\mathbf{x}) = \sum_{i,k} I_i \, \hat{p}_i^D \frac{\alpha_{i,k}\,\mathbf{x}_k}{\sum_{k'} \alpha_{i,k'} \hat{\mathbf{x}}_{k'}} \, T_k$.

Probabilities computation using MNL model

Using the utilities seen earlier, we can now build a Multinomial Logit (MNL) model for transportation mode choice, giving us the following transportation modes probabilities:

$$\begin{aligned}
p_{i,C}^B(\mathbf{w}, \mathbf{x}) &= \frac{\exp(u_{i,C}^W)}{\exp(u_i^S) + \exp(u_i^D) + \sum_{C' \in \mathcal{A}_i} \exp(u_{i,C'}^W)} \\
p_i^D(\mathbf{w}, \mathbf{x}) &= \frac{\exp(u_i^D)}{\exp(u_i^S) + \exp(u_i^D) + \sum_{C' \in \mathcal{A}_i} \exp(u_{i,C'}^W)} \\
p_i^S(\mathbf{w}, \mathbf{x}) &= \frac{\exp(u_i^S)}{\exp(u_i^S) + \exp(u_i^D) + \sum_{C' \in \mathcal{A}_i} \exp(u_{i,C'}^W)}
\end{aligned}$$

Except for neighborhoods 0 (outside nearby neighborhoods) and 1 (new on-site population) where $p_0^B(\mathbf{w}, \mathbf{x}) = 0$ and $p_1^D(\mathbf{w}, \mathbf{x}) = 0$.

Government's Problem

The government's objective is to decide on the optimal land-use distribution ($\mathbf{x}$) and the development of bike lane segments ($\mathbf{w}$) to maximize the site's total accessibility. This involves balancing the needs of residents who prefer biking with those who prefer driving, under budget constraints for both land area and bike lane improvements. The total accessibility to the site is defined as a barycenter of the bike objective g_B and the car objective g_D:

$$\begin{aligned}
g_B(\mathbf{w}, \mathbf{x}) &= \rho_1 \, \mathbf{x}_1 \sum_{C \in \mathcal{A}_1} p_{1,C}^B + \sum_{i > 2} I_i \sum_{C \in \mathcal{A}_i} p_{i,C}^B \\
g_D(\mathbf{x}, \mathbf{x}) &= I_0 \; p_0^D + \sum_{i > 2} I_i \; p_i^D
\end{aligned}$$

In the previous definitions, g_B is composed of two terms, the first one being the number of people living in the site residential area that are wishing to use the site's infrastructure and the second term represents the number of people living in nearby neighborhoods wishing to bike to the site.

The first term in g_D represents the people living outside the nearby neighborhoods wishing to come to the site while the second one is the equivalent for people from nearby neighborhoods.

Given these definitions we can now define the government's problem:

$$\begin{aligned}\max_{\mathbf{w},\mathbf{x}} \quad & \mu_B \; g_B(\mathbf{w},\mathbf{x}) + (1-\mu_B)\; g_D(\mathbf{w},\mathbf{x}) \\ \text{s.t.} \quad & \sum_{s\in\mathcal{S}} \mathbf{w}_s \, \|s\| \leq M^B \\ & \sum_k \mathbf{x}_k \leq M^A \\ & \mathbf{x} \geq 0 \\ & \mathbf{w} \in \{0,1,2\}^{|\mathcal{S}|}\end{aligned}$$

with $\mu_B \in [0,1]$ being the weight associated with the biking objective, the first constraint being the bike lane budget one and the second constraint being the site area constraint. As one of the project's objectives is to provide car alternatives, we even have $\mu_B > 0.5$.

Through this objective function the model's objective is plural. On the one side, the model wishes to increase the number of people visiting the site (by increasing g_B and g_D and therefore decreasing all p_i^S), on the over side the model prioritizes people biking to the site to those driving. Both objectives are made by increasing the site's attractiveness (through the term $\sum_{k\in\mathcal{K}} \alpha_{i,k}\,\mathbf{x}_k$ present in both $u_{i,C}^B$ and u_i^D) and by upgrading the cycling infrastructures (through the terms $+\lambda_2^B \sum_{l=1}^{n_C} \mathbf{w}_{s_C^l} \, \|s_C^l\| + \lambda_3^B \left(\sum_{l=1}^{n_C-1} \mathbb{1}(\mathbf{w}_{s_C^l}\,\mathbf{w}_{s_C^{l+1}} > 0)\right)\left(\frac{\|C\|}{n_C-1}\right)$ present in $u_{i,C}$ utilities).

3.2 Granular Model

In the granular model, we shift our focus from a holistic view to a detailed examination of the site as comprising multiple zones, each with its distinct area. This approach allows us to delve deeper into the site's internal architecture and accessibility, providing a more nuanced optimization strategy.

Let,

$$\begin{aligned}
\mathcal{J} &: \text{ the set of zones within the site} \\
\mathcal{A}_{i,j} &: \text{ the set of the } n^{th} \text{ shortest routes connecting } i \text{ to the zone } j, \\
\mathbf{x}_{j,k} &: \text{ the decision variable representing the area of zone } j \text{ used for land-use } k, \\
\|\tilde{C}^D_{i,j}\| &: \text{ the total length of the shortest driving path from } i \text{ to zone } j, \\
M^A_j &: \text{ the total area of zone } j, \text{ so that } M^A = \sum_j M^A_j.
\end{aligned}$$

Precise Destinations and Utilities

Our goal remains to maximize the total accessibility of the site, but now we consider residents' movements more explicitly. We begin by examining the behavior of a resident from neighborhood $i \in \mathcal{I}$. First, the resident decides on the type of activity they wish to engage in, such as visiting a park, grocery shopping, or going to a cafe. This decision is represented by the resident's desire to visit a type $k \in \mathcal{K}$ facility.

Once the resident has chosen an activity type k, they consider the various options available to them. Specifically, they look at each zone of the site $j \in \mathcal{J}$ where the land-use type k is present, indicated by $\mathbf{x}_{j,k} > 0$. The resident then evaluates their transportation choices.

They may choose to bike to destination j via a specific route $C \in \mathcal{A}_{i,j}$, with the utility associated to this option denoted by $u^B_{i,C,j,k}$. Alternatively, they may decide to drive to destination j, with a utility $u^D_{i,j,k}$. If none of the destinations j are suitable, they might opt for an outside alternative, with this choice's utility denoted by $u^S_{i,k}$.

We can now define explicitly these new utilities:

$$\begin{aligned}
u^B_{i,C,j,k} &= \alpha_{i,k}\, \mathbf{x}_{j,k} - \lambda^B_1 \|C\| + \lambda^B_2 \sum_{l=1}^{n_C} \mathbf{w}_{s^l_C} \|s^l_C\| + \lambda^B_3 \left(\sum_{l=1}^{n_C - 1} \mathbb{1}(\mathbf{w}_{s^l_C}\, \mathbf{w}_{s^{l+1}_C} > 0) \right) \left(\frac{\|C\|}{n_C - 1} \right) \\
u^D_{i,j,k} &= \alpha_{i,k}\, \mathbf{x}_{j,k} - \lambda^D_1 \|\tilde{C}^D_{i,j}\| - \lambda^D_2 f_P(\mathbf{x}) \\
u^S_{i,k} &= \beta^S = 0
\end{aligned}$$

with f_P having the same formula but now considering P_0 as proportional to $\sum_j \mathbf{x}_{j,\text{Parking}}$.

In both utilities $u^B_{i,C,j,k}$ and $u^D_{i,j,k}$ the first term represents the attractiveness of the single land-use type k while the other terms remain unchanged from the holistic model.

Choice Probabilities

Keeping the MNL model, the new choice probabilities are given by:

$$
\begin{aligned}
p^B_{i,C,j,k} =& \frac{\exp(u^B_{i,C,j,k})\,\mathbb{1}(\mathbf{x}_{j,k}>0)}{\exp(u^S_{i,k}) + \sum_{j'}\sum_{C'\in\mathcal{A}_{i,j'}}\exp(u^B_{i,C',j',k})\,\mathbb{1}(\mathbf{x}_{j',k}>0) + \sum_{j'}\exp(u^D_{i,j',k})\,\mathbb{1}(\mathbf{x}_{j',k}>0)}\\
p^D_{i,j,k} =& \frac{\exp(u^D_{i,j,k})\,\mathbb{1}(\mathbf{x}_{j,k}>0)}{\exp(u^S_{i,k}) + \sum_{j'}\sum_{C'\in\mathcal{A}_{i,j'}}\exp(u^B_{i,C',j',k})\,\mathbb{1}(\mathbf{x}_{j',k}>0) + \sum_{j'}\exp(u^D_{i,j',k})\,\mathbb{1}(\mathbf{x}_{j',k}>0)}\\
p^S_{i,k} =& \frac{\exp(u^S_{i,k})\,\mathbb{1}(\mathbf{x}_{j,k}>0)}{\exp(u^S_{i,k}) + \sum_{j'}\sum_{C'\in\mathcal{A}_{i,j'}}\exp(u^B_{i,C',j',k})\,\mathbb{1}(\mathbf{x}_{j',k}>0) + \sum_{j'}\exp(u^D_{i,j',k})\,\mathbb{1}(\mathbf{x}_{j',k}>0)}
\end{aligned}
$$

We can thus denote

$$
\begin{aligned}
p^B_{i,k} &= \sum_{j\in\mathcal{J}}\sum_{C\in\mathcal{A}_{i,j}} p^B_{i,C,j,k}\\
p^D_{i,k} &= \sum_{j\in\mathcal{J}} p^D_{i,j,k}
\end{aligned}
$$

as the probability of biking and driving to facility type k from location i.

Government's Problem

The government's problem keeps the same structure as the holistic one. The new bike and car objectives g_B and g_D are:

$$
\begin{aligned}
g_B(\mathbf{w},\mathbf{x}) &= \rho_1\,\mathbf{x}_1\sum_{k\in\mathcal{K}} p^B_{1,k} + \sum_{i>2} I_i \sum_{k\in\mathcal{K}} p^B_{i,k}\\
g_D(\mathbf{w},\mathbf{x}) &= I_0\sum_{k\in\mathcal{K}} p^D_{0,k} + \sum_{i>2} I_i \sum_{k\in\mathcal{K}} p^D_{i,k}
\end{aligned}
$$

We also want to incorporate two additional constraints into our model. The first constraint limits the total number of each land-use building to a certain maximal value (N_k for $k\in\mathcal{K}$), such as setting a maximum of four zones where grocery stores can be located. This ensures a balanced distribution of land-use types across the development site. The second constraint specifies a minimum area (m_k for $k\in\mathcal{K}$) requirement for each land-use type. For example, parks must occupy at least a certain minimum area, ensuring that these spaces are large enough to serve their intended purpose effectively.

The granular optimization problem is now formulated as follows:

$$
\begin{aligned}
\max_{\mathbf{w},\mathbf{x}} \quad & \mu_B \; g_B(\mathbf{w}, \mathbf{x}) + (1 - \mu_B) \; g_D(\mathbf{w}, \mathbf{x}) \\
\text{s.t.} \quad & \sum_{s \in \mathcal{S}} w_s \|s\| \leq M^B \\
& \sum_{k} x_{j,k} \leq M_j^A \qquad \forall j \in \mathcal{J} \\
& \sum_{j} \mathbb{1}(\mathbf{x}_{j,k} > 0) \leq N_k \qquad \forall k \in \mathcal{K} \\
& \mathbf{x}_{j,k} \geq m_k \, \mathbb{1}(\mathbf{x}_{j,k} > 0) \qquad \forall j,k \in \mathcal{J} \times \mathcal{K} \\
& \mathbf{x} \geq 0 \\
& \mathbf{w} \in \{0, 1, 2\}^{|\mathcal{S}|}
\end{aligned}
$$

with the second constraint being the area constraint for each zone $j \in \mathcal{J}$ within the development site.

In conclusion, the granular model offers a more nuanced and detailed approach compared to the holistic model by focusing on the internal structure of the development site. While most papers on urban redevelopment tend to overlook the inner-site architecture, our research has intentionally focused on this granular model. By delving into the detailed distribution of redevelopment sites, we aimed to innovate beyond traditional approaches. This detailed analysis allows us to address the complexities of urban spaces with a fresh perspective, potentially leading to more effective and innovative redevelopment strategies.

4 OPTIMIZATION PROCESS

This section presents two distinct methods for linearizing (we still allow indicator variables) the optimization model and a computational approach that reduces the problem's complexity. For clarity and ease of notation, the forthcoming discussion will focus on the holistic model in the specific case where we only consider the shortest bike route (such that $|\mathcal{A}_i| = 1 \quad \forall i$) and we will denote as u_i^B and p_i^B the corresponding utility and probability. However, the principles applied are directly transferable to the case where multiple route paths are considered and to the granular model.

4.1 Linearizarion Process

In both the following methods, we will be simplifying g_B by assuming $p_1^B \approx 1$. Indeed, it allows us to get rid of the bilenear term of g_B as the absence of driving option for the people living in the site makes p_1^B close to 1. Therefore,

$$g_B(\mathbf{w}, \mathbf{x}) \approx \rho_1 \, \mathbf{x}_1 + \sum_{i>2} I_i \; p_i^B$$

4.1.1 • A Linearization Using a Subproblem

In [Liu et al, 2022], a method is proposed to linearize a complex optimization problem involving discrete choice models. This approach, though tailored for a different context, provides a framework for linearizing our own problem. By adapting the solution methodology described in their paper, we apply similar linearization techniques to our problem, thereby simplifying the optimization process and making it more tractable for optimization solvers like Gurobi.

To streamline the problem, we start by reformulating it with the probabilities p as decision variables:

$$
\begin{aligned}
\max_{\mathbf{w},\mathbf{x},p} \quad & \mu_B \; g_B(\mathbf{w},\mathbf{x}) + (1-\mu_B)\; g_D(\mathbf{w},\mathbf{x}) \\
\text{s.t.} \quad & \sum_{s\in\mathcal{S}} w_s \|s\| \le M^B \\
& \sum_k x_k \le M^A \\
& \mathbf{x} \ge 0 \\
& \mathbf{w} \in \{0,1,2\}^{|\mathcal{S}|} \\
& p \ge 0 \\
& \sum_{m\in\mathcal{M}} p_i^m = 1 \qquad \forall i \in \mathcal{I} \\
& p_i^m = \frac{e^{u_i^m}}{\sum_{m'} e^{u_i^{m'}}} \qquad \forall i,m \in \mathcal{I}\times\mathcal{M}
\end{aligned}
$$

Our goal is to ensure that the last three constraints are satisfied. To achieve this, we introduce the function:

$$
S(p,\mathbf{x},\mathbf{w}) = -\sum_{i,m} u_i^m p_i^m + \sum_{i,m} p_i^m \log(p_i^m)
$$

The Lagrangian for S, given constraints $p \ge 0$ and $\sum_m p = 1$, is:

$$
\mathcal{L}(p,\mathbf{x},\mathbf{w}) = -\sum_{i,m} u_i^m p_i^m + \sum_{i,m} p_i^m \log(p_i^m) - \sum_i a_i^m p_i^m + \sum_i b_i \left(\sum_m p_i^m - 1\right)
$$

where $a_i^m \ge 0$. Applying the complementary slackness condition and stationary conditions, we derive:

$$
\begin{cases}
a_i^m p_i^m & = 0 \qquad \forall i,m \\
\frac{\partial \mathcal{L}}{\partial p_i^m} & = -u_i^m + \log(p_i^m) + 1 - \lambda_i^m + \mu_i = 0 \qquad \forall i,m
\end{cases}
$$

For any feasible solution, we find that $a_i^m = 0$ and:

$$
p_i^m = \frac{e^{u_i^m}}{e^{1+b_i}}
$$

with $\frac{1}{e^{1+b_i}} = \frac{1}{\sum_{m'} e^{u_i^{m'}}}$, which implies that the solution satisfies the Multinomial Logit (MNL) choice model.

Thus, we can rewrite the problem as:

$$\begin{aligned}
\max_{\mathbf{w},\mathbf{x},p} \quad & \mu_B \; g_B(\mathbf{w},\mathbf{x}) + (1-\mu_B) \; g_D(\mathbf{w},\mathbf{x}) \\
\text{s.t.} \quad & \sum_{s\in\mathcal{S}} w_s \|s\| \leq M^B \\
& \sum_k x_k \leq M^A \\
& \mathbf{x} \geq 0 \\
& \mathbf{w} \in \{0,1,2\}^{|\mathcal{S}|} \\
& (p,\mathbf{x},\mathbf{w}) = \underset{p,\mathbf{x},\mathbf{w}}{\operatorname{argmin}} \, S(p,\mathbf{x},\mathbf{w}) \\
& \qquad \text{s.t.} \qquad p \geq 0 \\
& \qquad \qquad \qquad \sum_m p_i^m = 1 \qquad \forall i
\end{aligned}$$

Approximation of $u.p$ and $p.log(p)$

To approximate S into a linear function we have to approximate its two terms $u.p$ and $p.log(p)$.

As commercial optimization solvers allows implications from binary variables we can easily fix the first term $u.p$ by discretizing $\mathbf{x}$ (into percentages of M^A) and therefore u, as $\mathbf{w}$ is already discrete. Let $u^n, \quad n \leq N$ the possible values for u, we can now define $u.p$ through the following implications:

$$\forall n \leq N, \qquad u = u^n \implies u.p = u^n.p$$

To linearize $p_i^m \log(p_i^m)$, we want to approximate the function using its tangents, for $r \leq R \in \mathbb{N}$, the r^{th} tangent is given by the equation:

$$\psi_{i,r}(p) = (1 + \log(p_{i,r}))p - p_{i,r}$$

with $p_{i,r} \in (0,1]$ the contact point with the tangent. Consequently, we consider a variable ψ_i^m that will approximate $p_i^m \log(p_i^m)$ by $\max_{1\leq r\leq R} \psi_{i,r}(p_i^m)$.

For any $R \in \mathbb{N}$, we choose all $p_{i,r}$ in order to minimize the quantity

$$\varepsilon_R = \sup_{p\in[0,1]} \max_{r\in argmax\{\psi_{i,r}(p)\}} |(1+\log(p)) - (1+\log(p_{i,r}))|$$

In fact, as shown in the appendix, we can show that ε_R is responsible for the quality of the approximation through $e^{-2\varepsilon_R}$ and $e^{2\varepsilon_R}$:

$$\frac{e^{u_i^m}}{\sum_{m'} e^{u_i^{m'}}} \cdot e^{-2\varepsilon_R} \leq \; p_i^m \; \leq \frac{e^{u_i^m}}{\sum_{m'} e^{u_i^{m'}}} \cdot e^{2\varepsilon_R}$$

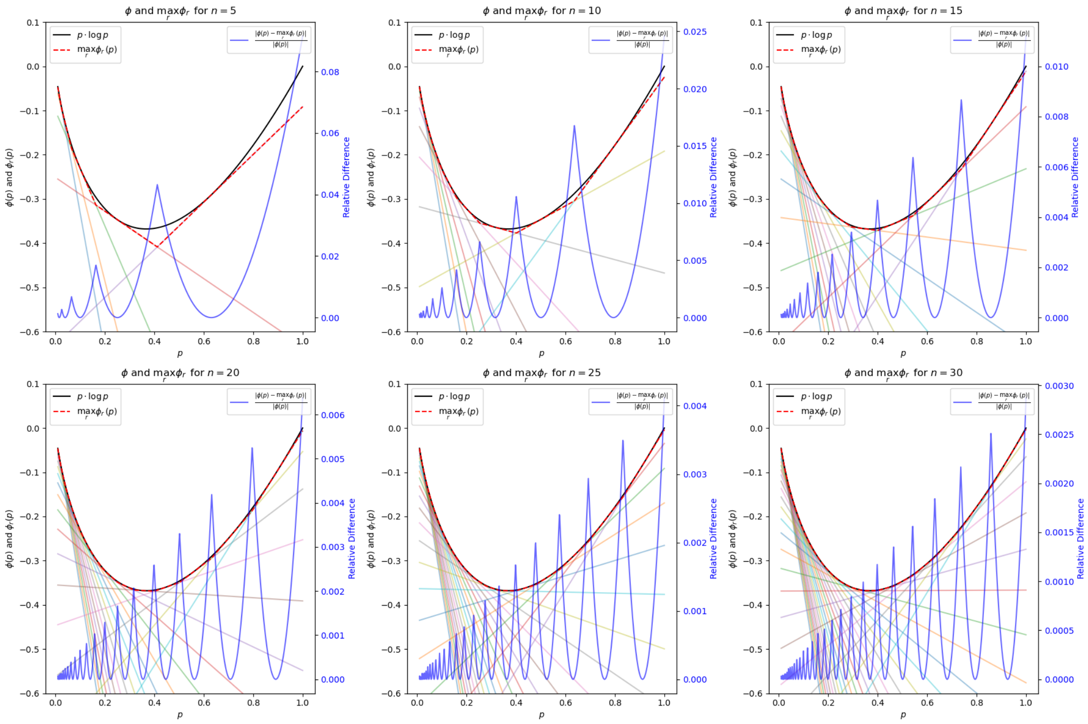

Figure 1: $p \cdot \log(p)$ and $max_r \psi_{i,r}(p)$

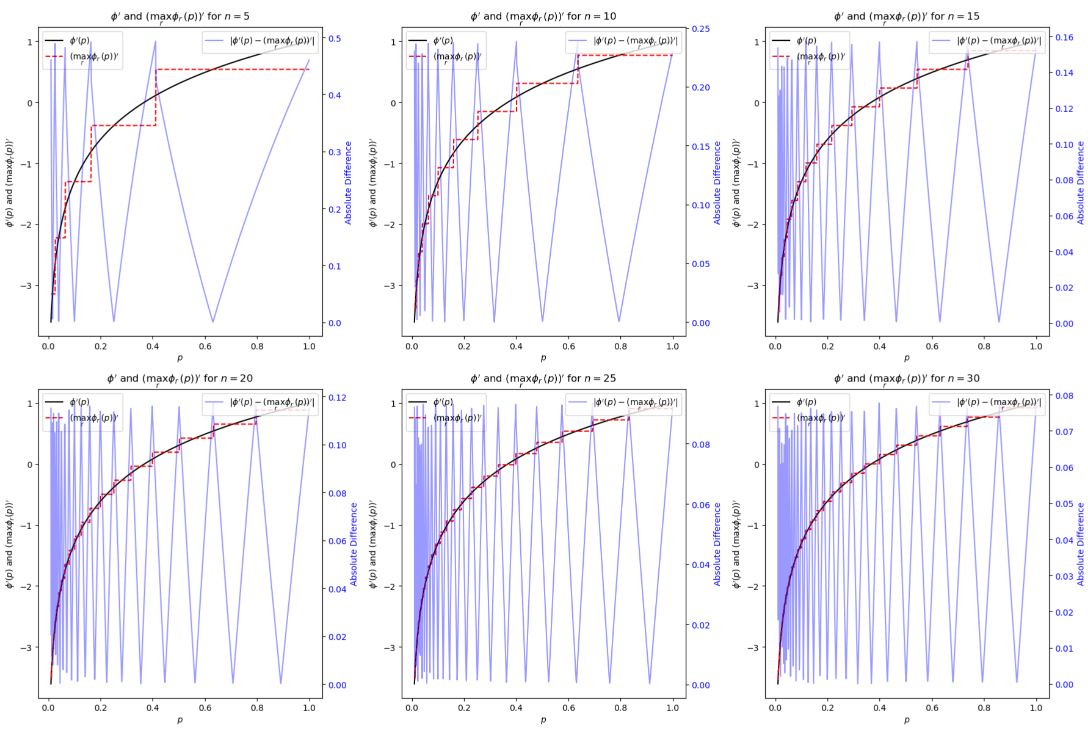

Figure 2: $1 + \log(p)$ and $(max_r \psi_{i,r}(p))'$

Therefore, the following table provides estimations of the maximal potential errors with these approximations:

R	5	10	15	20	25	30
$1-e^{-2\varepsilon}$	0.60	0.37	0.26	0.21	0.17	0.14
$e^{2\varepsilon}-1$	1.51	0.58	0.36	0.26	0.20	0.17

Table 1: Values of $e^{-2\varepsilon}$ and $e^{2\varepsilon}$ for different R

Linear reformulation

Defining the function:

$$S_L(p, \mathbf{x}, \mathbf{w}, \psi) = -\sum_{i,m} u_i^m p_i^m + \sum_{i,m} \phi_i^m$$

and its Lagrangian associated to to the constraints $p \geq 0$, $\forall i,\ \sum_m p_i^m = 1$ and $\forall i, r, m,\ \psi_i^m \geq \psi_{i,r}(p_i^m)$:

$$\mathcal{L}_L(p, \mathbf{x}, \mathbf{w}, \psi) = -\sum_{i,m} u_i^m p_i^m + \sum_{i,m} \phi_i^m - \sum_{i,m} a_i^m p_i^m + \sum_i b_i \left(\sum_m p_i^m - 1\right) - \sum_{i,m,r} \gamma_{i,r}^m (\psi_i^m - \psi_{i,r}(p_i^m))$$

with $a, \gamma \geq 0$, the appendix shows that the following problem's solution is a $e^{2\varepsilon}$ estimation of the exact solution:

$$\begin{aligned}
\max_{\mathbf{w},\mathbf{x},p,\psi} \quad & \mu_B\ g_B(\mathbf{w},\mathbf{x}) + (1-\mu_B)\ g_D(\mathbf{w},\mathbf{x}) \\
\text{s.t.} \quad & \sum_{s\in\mathcal{S}} w_s \|s\| \leq M^B \\
& \sum_k x_k \leq M^A \\
& \mathbf{x} \geq 0 \\
& \mathbf{w} \in \{0,1,2\}^{|\mathcal{S}|} \\
& (p,\mathbf{x},\mathbf{w}) = \operatorname*{argmin}_{p,\mathbf{x},\mathbf{w}} S_L(p,\mathbf{x},\mathbf{w}) \\
& \qquad \text{s.t.} \qquad p \geq 0 \\
& \qquad\qquad\qquad \sum_m p_i^m = 1 \qquad \forall i \\
& \qquad\qquad\qquad \psi_i^m \geq \psi_{i,r}(p_i^m) \qquad \forall i, m, r
\end{aligned}$$

Now we can replace the argmin constraint by using strong duality (optimal solutions can be described as all $p, \mathbf{x}, \mathbf{w}, \psi, a, b, \gamma$ such that $p, \mathbf{x}, \mathbf{w}, \psi$ is primal feasible and a, b, γ is dual feasible).

We can express the dual's version of the Lagrangian as follows:

$$\begin{aligned}\mathcal{L}_L(p,\psi,y,w,P_0) = & -\sum_i b_i - \sum_{i,r,m} \gamma_{i,r}^m p_{i,r} + \sum_{i,m} p_i^m \left(-u_i^m - a_i^m + b_i + \sum_r \gamma_{i,r,m}(1+\log(p_{i,r}))\right) \\ & + \sum_{i,m} \psi_i^m \left(1 - \sum_r \gamma_{i,r,m}\right)\end{aligned}$$

with $p_i^m \geq 0$.

So our final linear approximation is:

$$\begin{aligned}\max_{\mathbf{w},\mathbf{x},p,\psi,a,b,\gamma} \quad & \mu_B \; g_B(\mathbf{w},\mathbf{x}) + (1-\mu_B)\; g_D(\mathbf{w},\mathbf{x}) \\ \text{s.t.} \quad & \sum_{s\in\mathcal{S}} w_s \|s\| \leq M^B \\ & \sum_k x_k \leq M^A \\ & \mathbf{x} \geq 0 \\ & \mathbf{w} \in \{0,1,2\}^{|\mathcal{S}|} \\ & S_L(p,\mathbf{x},\mathbf{w}) = -\sum_i b_i - \sum_{i,r,m} \gamma_{i,r}^m p_{i,r} && \text{(Strong Duality)} \\ & p \geq 0 && \text{(Primal constraints)} \\ & \sum_m p_i^m = 1 \qquad \forall i \in \mathcal{I} \\ & \psi_i^m \geq \psi_{i,r}(p_i^m) \qquad \forall i,m,r \\ & -u_i^m - a_i^m + b_i + \sum_r \gamma_{i,r}^m (1+\log(p_{i,r})) \geq 0 \quad \forall i,m && \text{(Dual constraints)} \\ & \sum_r \gamma_{i,r}^m = 1 \qquad \forall i,m\end{aligned}$$

Making both the number of variables and constraints in $\mathcal{O}(|\mathcal{S}| + |\,\mathcal{I}\,| \cdot |\,\mathcal{M}\,| \cdot R)$.

4.1.2 • A Linearization by 2D Piecewise Approximation of the Sigmoid Function

In this section, we explore another approach to linearizing our model, inspired by the work of [Cardaso et al, 2017]. This method involves creating a piecewise approximation of the objective constraint using Special Ordered Sets of type 2 (SOS2) constraints. SOS2 constraints require that a vector has at most two non-zero elements, and if there are two non-zero elements, they must be consecutive.

The basic use of an SOS2 is to transform a continuous value into a barycentre of two discrete values. Lets consider a continuous variable u between 0 and 10. Then we can define $(\lambda_i)_{0\leq i\leq 10}$ a SOS2 positive vector of sum 1 and define the constraint

$$u = \sum_{0\leq i\leq 10} \lambda_i \cdot i$$

Therefore, the only non-zeros values of λ will be $\lambda_{\lfloor u\rfloor}$ and $\lambda_{\lceil u\rceil}$ and we can estimate in u any non-linear function f by:

$$f(u) \approx \sum_{0\leq i\leq 10} \lambda_i \cdot f(i)$$

In our case, we will employ a two-dimensional positive vector that sums to 1, with each dimension satisfying an SOS2 constraint. This approach is akin to dividing a 2D plane into a grid and treating each point as the barycenter of the four corners surrounding it. Specifically, the two dimensions will represent u_B and u_D. By doing so, we can approximate the Multinomial Logit (MNL) model using the barycentric coordinates of the discrete MNL values.

Indeed, let define $(\tilde{u}^B_{j^B})_{j^B\leq N}$ and $(\tilde{u}^D_{j^D})_{j^D\leq N}$ some discrete values that divide the intervals of possibles values for all u^B_i and u^D_i into $N \in \mathbb{N}$ values and:

$$f^B_{\text{MNL}}(x,y) = \frac{\exp(x)}{1 + exp(x) + exp(y)}$$
$$f^D_{\text{MNL}}(x,y) = \frac{\exp(y)}{1 + exp(x) + exp(y)}$$

For a certain $i \in \mathcal{I}$, lets denote $\lambda^i_{j^B,j^D}$ our 2D positive vector of sum 1. We can now approximate the MNL model with the following constraints:

$$\begin{aligned}
(\lambda^i_{j^B,j^D})_{j^B \le N} \text{ satisfies a SOS2 constraint} \qquad & \forall j^D \le N \\
(\lambda^i_{j^B,j^D})_{j^D \le N} \text{ satisfies a SOS2 constraint} \qquad & \forall j^B \le N \\
u_i^B = \sum_{j^B,j^D} \lambda^i_{j^B,j^D} \tilde{u}^B_{j^B} & \\
u_i^D = \sum_{j^B,j^D} \lambda^i_{j^B,j^D} \tilde{u}^D_{j^D} & \\
p_i^B = \sum_{j^B,j^D} \lambda^i_{j^B,j^D} f^B_{\text{MNL}}(\tilde{u}^B_{j^B}, \tilde{u}^D_{j^D}) & \\
p_i^D = \sum_{j^B,j^D} \lambda^i_{j^B,j^D} f^D_{\text{MNL}}(\tilde{u}^B_{j^B}, \tilde{u}^D_{j^D}) &
\end{aligned}$$

So our final linear approximation is:

$$\begin{aligned}
\max_{\mathbf{w},\mathbf{x},p,\lambda} \quad & \mu_B \; g_B(\mathbf{w},\mathbf{x}) + (1-\mu_B) \; g_D(\mathbf{w},\mathbf{x}) \\
\text{s.t.} \quad & \sum_{s \in \mathcal{S}} w_s \|s\| \le M^B \\
& \sum_k x_k \le M^A \\
& \mathbf{x} \ge 0 \\
& \mathbf{w} \in \{0,1,2\}^{|\mathcal{S}|} \\
& \lambda \ge 0 \\
& \sum_{j^B,j^D} \lambda^i_{j^B,j^D} = 1 \qquad \forall i \in \mathcal{I} \\
& (\lambda^i_{j^B,j^D})_{j^B \le N} \text{ satisfies a SOS2 constraint} \qquad \forall i \in \mathcal{I}, j^D \le N \\
& (\lambda^i_{j^B,j^D})_{j^D \le N} \text{ satisfies a SOS2 constraint} \qquad \forall i \in \mathcal{I}, j^B \le N \\
& u_i^B = \sum_{j^B,j^D} \lambda^i_{j^B,j^D} \tilde{u}^B_{j^B} \qquad \forall i \in \mathcal{I} \\
& u_i^D = \sum_{j^B,j^D} \lambda^i_{j^B,j^D} \tilde{u}^D_{j^D} \qquad \forall i \in \mathcal{I} \\
& p_i^B = \sum_{j^B,j^D} \lambda^i_{j^B,j^D} f^B_{\text{MNL}}(\tilde{u}^B_{j^B}, \tilde{u}^D_{j^D}) \qquad \forall i \in \mathcal{I} \\
& p_i^D = \sum_{j^B,j^D} \lambda^i_{j^B,j^D} f^D_{\text{MNL}}(\tilde{u}^B_{j^B}, \tilde{u}^D_{j^D}) \qquad \forall i \in \mathcal{I}
\end{aligned}$$

Making both the number of variables and constraints in $\mathcal{O}(|\mathcal{S}| + |\mathcal{I}| \cdot (|\mathcal{M}| + N^2))$.

Estimation Error of the Linearization

The estimation error of this linearization can be analyzed using Taylor's formula in two dimensions. Indeed, we can easily show that all the first-order terms sum to zero, leaving only the second-order terms. This results in a much better approximation than if we didn't use the barycenters but only the closest discrete points. The use of barycenters effectively reduces the approximation error, as the second-order terms provide a more accurate representation of the local behavior of the function near the points of interest.

This accuracy is reflected in the error values shown in the table below, where both the maximum error and relative errors decrease significantly as the number of points N increases, highlighting the benefits of this approach.

N	Max Error of p	Mean Relative Error	Max Relative Error
10	0.2961	0.2487	2.6287
20	0.1534	0.1089	0.8519
30	0.0969	0.0719	0.4983
50	0.0590	0.0414	0.2755
100	0.0286	0.0202	0.1257

Table 2: Estimation Errors for Different Values of N

4.2 Coordinate Descent Process made possible by the City's Architecture

The optimization problem can be effectively decoupled due to the site's limited number of entry points, which consist of only a few road segments that touch the site. This architecture allows us to simulate artificial neighborhoods at these entry points, breaking down the problem into two manageable subproblems:

- **Subproblem 1:** Optimizing the bike lane network and the number of people reaching the entry points.
- **Subproblem 2:** Applying the detailed model to the artificial neighborhoods defined by the entry points.

The coordinate descent algorithm can then be employed to iteratively solve these subproblems, alternating between them until an equilibrium is reached. This method leverages the solutions of one subproblem to inform and optimize the other, ensuring a coherent and integrated solution.

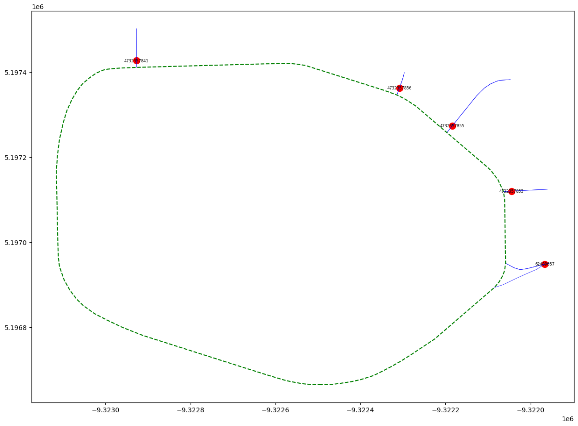

Figure 3: A schematic view of Briarwood Mall and its entry points

Figure 4: A satellite view of Briarwood Mall and its entry points

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Algorithm 1 Coordinate Descent Algorithm

Initialize: Set initial values $\mathbf{w}^0$ and $\mathbf{x}^0$ for bike lane developments and land-use areas.

Set: Maximum number of iterations T and tolerance ε.

for $t = 0$ to $T - 1$ **do**

Solve Subproblem 1: Optimize bike lanes network and compute entry point populations:

$$\mathbf{w}^{t+1} \leftarrow \arg\max_{\mathbf{w}} \ \mu_B \ g_B(\mathbf{w}, \mathbf{x}^t) + (1 - \mu_B) \ g_D(\mathbf{w}, \mathbf{x}^t)$$

Solve Subproblem 2: Optimize the site's land-use distribution using the entry points as neighborhoods:

$$\mathbf{x}^{t+1} \leftarrow \arg\max_{\mathbf{x}} \ \mu_B \ g_B(\mathbf{w}^{t+1}, \mathbf{x}) + (1 - \mu_B) \ g_D(\mathbf{w}^{t+1}, \mathbf{x})$$

Check Convergence: If $\| \mathbf{w}^{t+1} - \mathbf{w}^t \| < \varepsilon$ and $\| \mathbf{x}^{t+1} - \mathbf{x}^t \| < \varepsilon$, terminate the algorithm.

end for

Output: Optimal bike lane developments $\mathbf{w}^*$ and land-use areas $\mathbf{x}^*$.

In this algorithm, Subproblem 1 focuses on maximizing the site's total accessibility by determining the optimal bike lane network and the associated populations reaching the entry points. Subproblem 2 then takes these populations as fixed inputs and optimizes the detailed land-use model for the artificial neighborhoods defined by the entry points. The iterative nature of the coordinate descent ensures that both aspects are continuously refined until an equilibrium is reached, providing a solution to the overall optimization problem. However, it might be possible that the optimal solution found by the algorithm is not optimal with respect to the total detailed model.

5
CASE STUDY: BRIARWOOD MALL, MICHIGAN

The redevelopment of Briarwood Mall in Ann Arbor represents a significant transformation aimed at revitalizing and optimizing land use within the existing commercial space. This project is driven by multiple objectives, including enhancing commercial offerings, increasing residential availability, and improving transportation infrastructure, particularly for non-motorized transit.

Briarwood Mall, originally established in the early 1970s, has been a central retail hub in Ann Arbor. However, evolving market dynamics and community needs have necessitated a comprehensive redevelopment plan. The primary objectives of this redevelopment focus on diversifying land use, transitioning from a purely commercial space to a mixed-use development that includes residential units, retail spaces, and recreational areas. The proposal features 354 multi-family residential units alongside new commercial establishments, such as a two-level grocery store and a sporting goods store with an adjacent playing field. Sustainability and modernization are key aspects of the project, with the new residential building planned to be all-electric, supporting the city's environmental goals, and including various sustainability features. Enhancing connectivity and accessibility is also a major focus, with improvements to pedestrian and bicycle infrastructure aimed at making the mall more accessible. This includes new sidewalks, bike lanes, and reduced traffic lanes to enhance non-motorized access to the site, notably through the addition of a dedicated 5-foot-wide bike lane on Briarwood Circle, separated by a 2.5-foot painted stripe.

The redevelopment of Briarwood Mall aligns closely with the overarching goals of optimizing land use within the mall area and enhancing the bike lane network around it. By rezoning certain areas from parking to commercial and integrating residential spaces within the mall's footprint, the redevelopment utilizes land more efficiently, reducing expansive surface parking areas and replacing them with structures that add value and functionality, such as multi-level parking garages and landscaped areas. Promoting active transportation through significant improvements to the non-motorized transportation infrastructure addresses the need for safer and more convenient access for pedestrians and cyclists, supporting a healthier lifestyle and aligning with sustainable urban development practices. The proposed internal connector roadways and collaboration with the Ann Arbor Area Transportation Authority (AAATA) to integrate bus routes through the new development enhance the overall connectivity of the area, ensuring that the mall redevelopment supports broader city planning initiatives, including improved public transit and reduced vehicular traffic.

This case study involves exploring potential solutions for the redevelopment of the mall. Consequently, we considered the road and bike lane network of Ann Arbor as our network to be optimized and Briarwood Mall as the site to be transformed with mixed-use development. By focusing on these elements, the Briarwood Mall redevelopment project exemplifies a forward-thinking approach to urban planning that prioritizes sustainable growth, improved accessibility, and optimal land use by rejuvenating commercial areas and addressing contemporary environmental and social needs.

5.1 Data Collection

For the purpose of this case study, geographic data, including information on roads, bike lanes, and land areas, were sourced from OpenStreetMap. This data was accessed and collected using the Python package `osmnx`, which provides a convenient and efficient method for retrieving and manipulating spatial data.

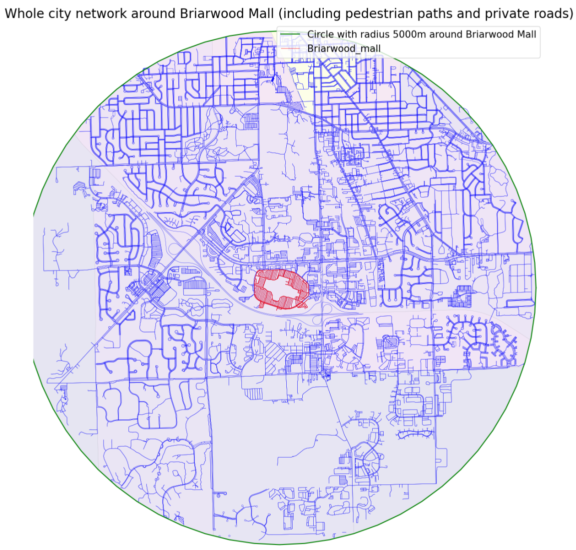

Figure 5: The whole city network around Briarwood Mall

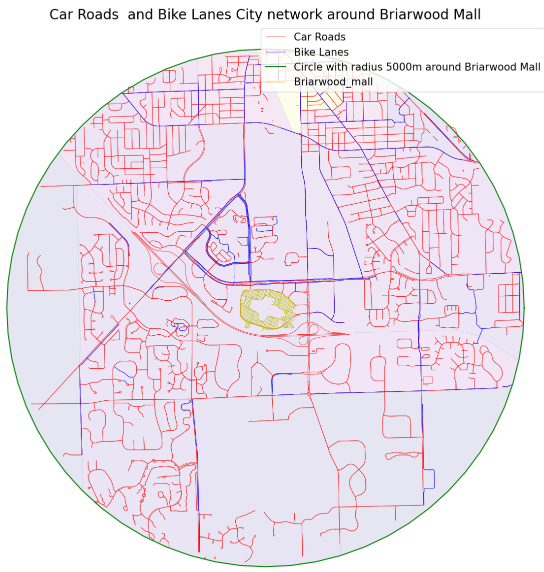

Figure 6: Cars and Bikes infrastructures around Briarwood Mall

In addition to geographic data, population density data for various locations within Ann Arbor was collected from the US Census Bureau. This data was retrieved using the Python package `cenpy`, which facilitates access to census data and supports detailed demographic analysis. By leveraging cenpy, we were able to gather precise population density figures and therefore to understand the distribution of residents around Briarwood Mall.

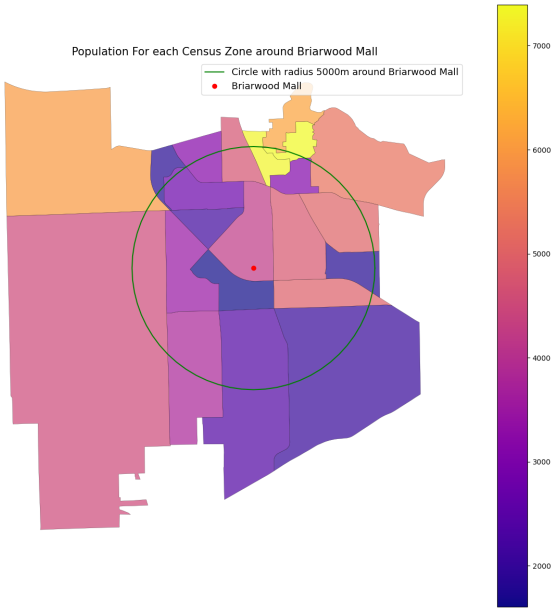

Figure 7: The census zones around Briarwood Mall

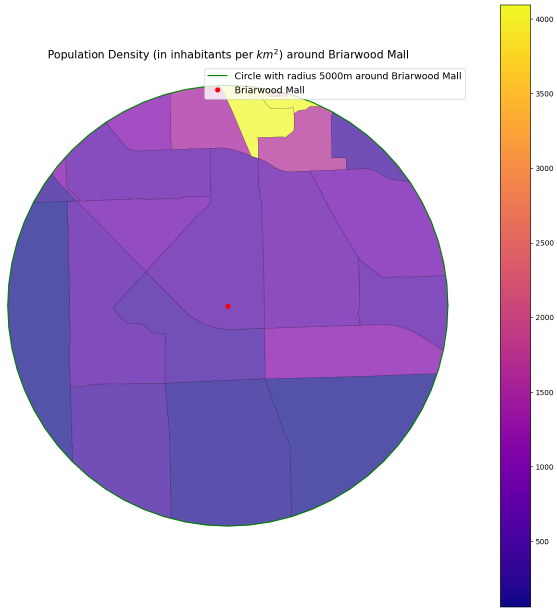

Figure 8: The population density around Briarwood Mall

5.2 Preprocessing

The preprocessing phase involves several critical steps aimed at preparing the data for detailed analysis and optimization. Initially, car roads and bike lanes have to be identified from the geographic data obtained through various filters. This step ensures a clear distinction between different types of transportation infrastructure.

Next, the nodes in the city network, originally representing every intersection of road segments, are clustered to simulate neighborhoods. This clustering process simplifies the network by grouping nearby intersections, effectively creating larger, more manageable units that represent different parts of the city. Using the population density data from the US Census Bureau, we then compute the estimated population for each of these neighborhoods, providing a demographic context for each cluster.

Simultaneously, the mall area is divided into multiple zones as seen in the Granular Model 3.2. The goal is to create zones of equivalent area where possible, with boundaries designed as straight lines to facilitate the creation of potential roads, paths, and bike lanes that connect the zones.

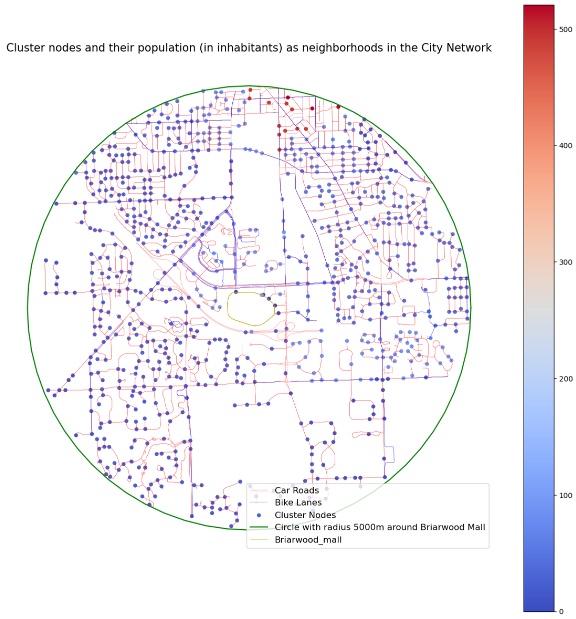

Figure 9: Cluster Nodes and their Population

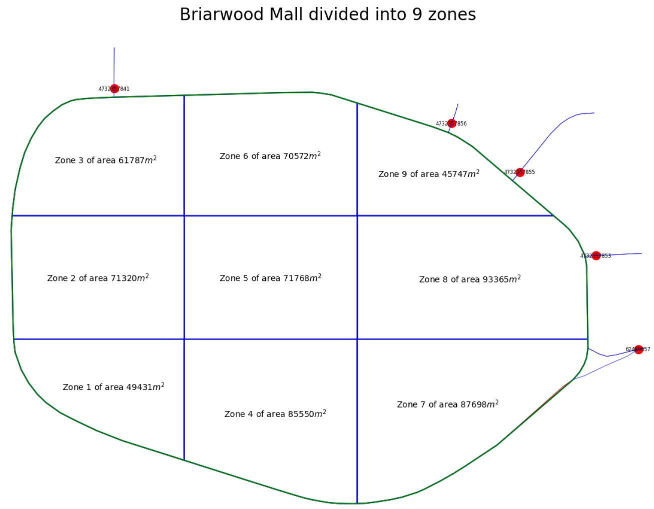

Figure 10: Division of Briarwood Mall into 9 zones

The final step of preprocessing involves implementing a Dijkstra algorithm, with each zone treated as a source vertex. This algorithm is used to compute the shortest paths connecting each neighborhood (cluster) to each zone within the mall site.

Algorithm 2 Dijkstra's Algorithm

Graph $G = (V, E)$, source vertex s

Shortest path distances from s to all vertices in V

Initialize distance to source: $dist[s] \leftarrow 0$

Initialize distances to all other vertices: $dist[v] \leftarrow \infty$ for all $v \in V \setminus \{s\}$

Initialize priority queue $Q \leftarrow \{(s, 0)\}$

Initialize previous node array: $prev[v] \leftarrow$ null for all $v \in V$

while Q is not empty **do**

    $u \leftarrow \textsc{ExtractMin}(Q)$

    for each neighbor v of u **do**

        $alt \leftarrow dist[u] + \text{length}(u, v)$

        if $alt < dist[v]$ **then**

            $dist[v] \leftarrow alt$

            $prev[v] \leftarrow u$

            $\textsc{InsertWithPriority}(Q, v, dist[v])$

        end if

    end for

end while

return $dist, prev$

5.3 Results

All results presented in this section were obtained using the granular model and the SOS2 linearization method, solved with the commercial solver Gurobi. The SOS2 linearization was selected due to its favorable computation time compared to other linearization techniques. Coordinate descent was not utilized as the computation time (from minutes to one hour) was deemed reasonable for the SOS2 approach.

The analysis of bike lane upgrades revealed that varying the priority factor μ for bicycles had minimal impact on the overall results. This suggests that the improvements to the bike lane infrastructure were effective regardless of the priority given to bicycles, demonstrating the robustness of the bike lane enhancements in the model.

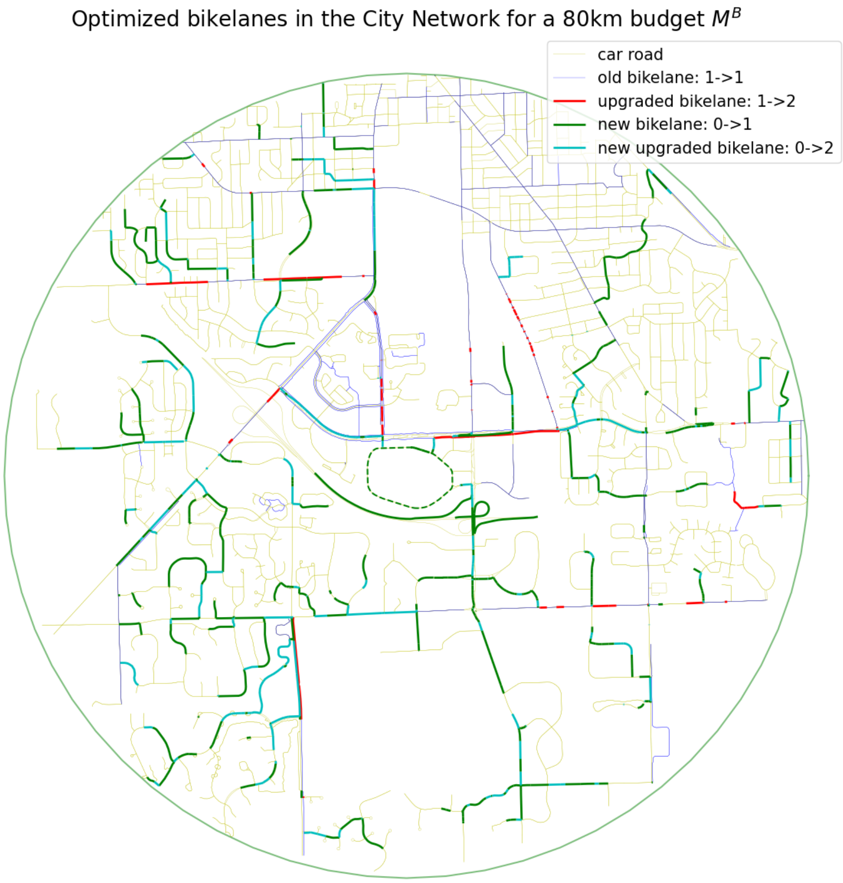

Figure 11: Optimized bike lane network for a 80km budget M^B

The redevelopment site was divided into nine distinct zones, with horizontal bars representing the allocation of each zone. For example, the northwestern zone with $\mu = 0.35$ shows a nearly uniform distribution across different land uses, except for residential areas. The results indicate a tendency to allocate non-residential land uses in zones that are more accessible from the main entry points, aligning with a logical approach to maximize accessibility. The gray bars in the visualizations represent the areas designated for roads, illustrating the integration of transportation infrastructure within the site layout.

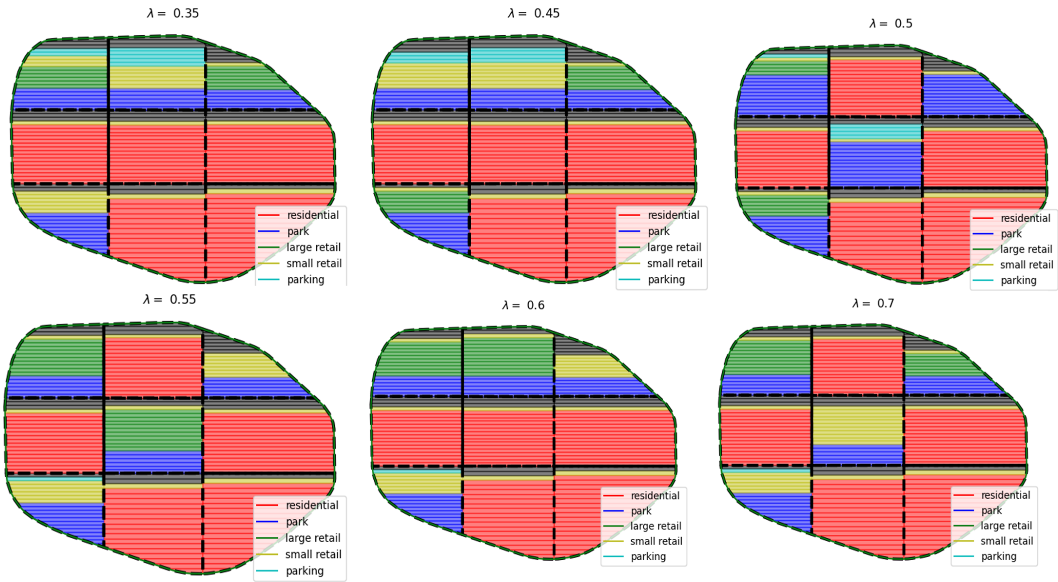

Figure 12: Distribution of the site for different values of μ

The optimal values computed by Gurobi were analyzed with respect to the parameter μ. The values of interest appeared to be between 0.45 and 0.55. A higher μ value (greater than 0.5) is preferred to promote biking. The fact that cars do not become more attractive when $\mu \ll 0.5$ is counterintuitive but can be attributed to the fact that the bike objective incorporates the entire population of new residents, balancing the attractiveness of biking and driving.

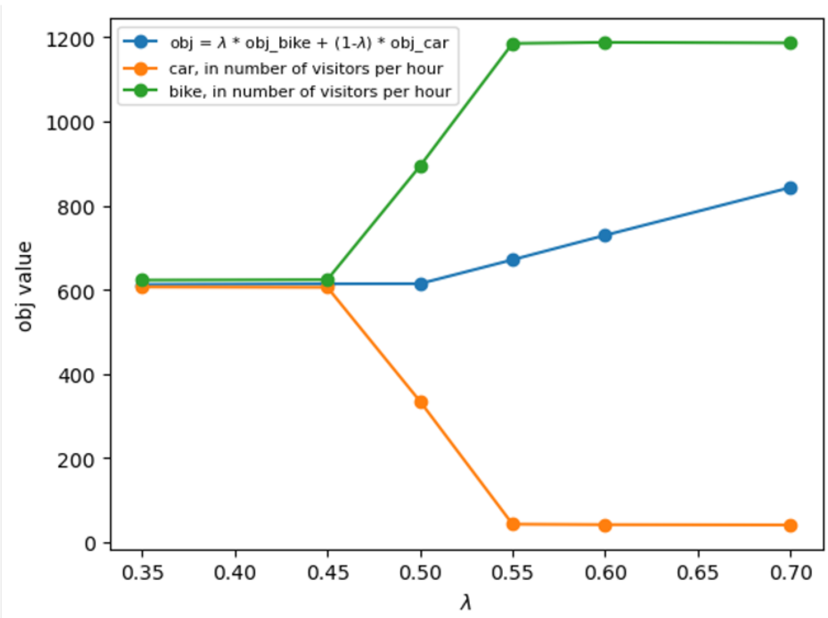

Figure 13: Objective values depending on μ

Additionally, the mean probability of using a car or bike, depending on μ and k (where $k = 1, 2, 3, 4$ represents residential, parks, large retail, and small retail, respectively), was examined. The results show that cars retain their advantage until μ reaches approximately 0.52, beyond which biking becomes relatively more attractive. This transition point highlights the sensitivity of transportation preferences to changes in μ, reflecting the dynamic interplay between biking and driving preferences.

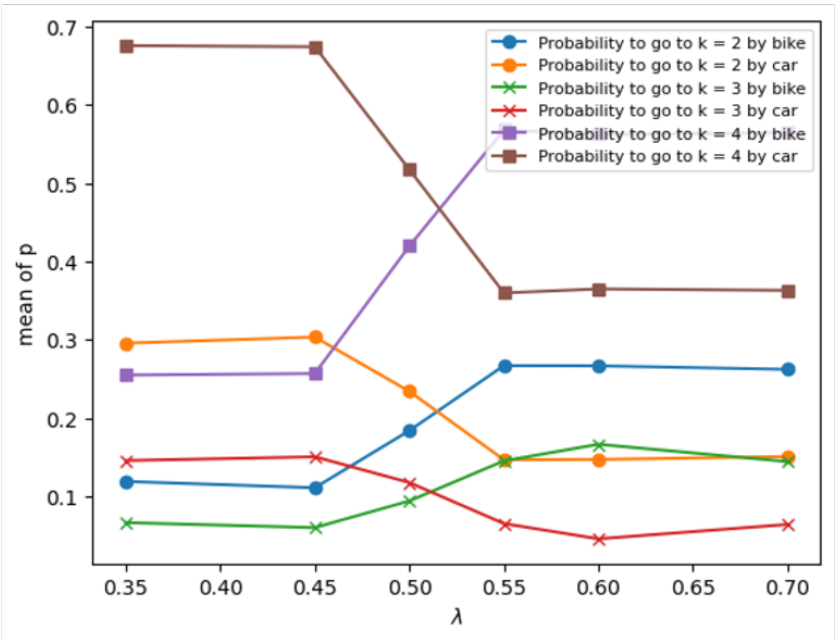

Figure 14: Probabilities depend on μ

6
CONCLUSION

The model developed in this study, particularly the granular model, presents an innovative and interesting approach to urban redevelopment. It stands out by enabling the analysis of interactions between the broader transportation network and the detailed infrastructure within the redevelopment site. This dual focus allows for a comprehensive understanding of both external connectivity and internal site optimization, facilitating a deeper dive into the internal layout and functionality of the site.

The preliminary results obtained from this model offer valuable insights, though they should be interpreted with caution. The findings demonstrate that a different approach to traditional urban redevelopment challenges can reveal new perspectives and features. By shifting focus to aspects such as non-motorized transit and mixed land-use, the study illustrates the potential benefits of modern urban planning practices.

One of the critical factors influencing the reliability of these results is the accuracy of the coefficients α, which represent the attractiveness of each land use. Currently, these coefficients are derived from surveys indicating how frequently people visit different land uses. To enhance the model's precision, obtaining more accurate and detailed data for these coefficients is essential. This would provide a more reliable basis for understanding land-use attractiveness and optimizing site design accordingly.

Looking ahead, the next significant step for improving the project's outcomes involves optimizing the internal routes within the site. Ensuring that all land uses are well-connected will facilitate easy movement for people, thereby enhancing the overall functionality and accessibility of the redevelopment. This optimization will contribute to creating a cohesive and user-friendly environment where residents and visitors can effortlessly navigate between different areas.

Another intriguing feature to consider adding to the model is the interaction between various land uses. For example, it would be beneficial to simulate whether visitors to large retail stores are likely to be induced to visit smaller retail outlets, even if they had not initially planned to do so. Incorporating such interactions would require precise data to support these assumptions, but it would add a valuable layer of realism to the model, further enhancing its utility and applicability.

In conclusion, while the current results are preliminary, they underscore the potential of innovative modeling approaches in urban redevelopment. By refining data inputs and expanding the model to include internal route optimization and land-use interactions, future research can build on these foundations to create even more effective and sustainable urban environments.

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APPENDIX

Estimation of the Approximation's Error of 4.1.1

Lets consider the function:

$$S_L(p, \mathbf{x}, \mathbf{w}, \psi) = -\sum_{i,m} u_i^m p_i^m + \sum_{i,m} \phi_i^m$$

and its Lagrangian associated to to the constraints $p \geq 0$, $\forall i,\ \sum_m p_i^m = 1$ and $\forall i, r, m,\ \psi_i^m \geq \psi_{i,r}(p_i^m)$:

$$\mathcal{L}_L(p, \mathbf{x}, \mathbf{w}, \psi) = -\sum_{i,m} u_i^m p_i^m + \sum_{i,m} \phi_i^m - \sum_{i,m} a_i^m p_i^m + \sum_i b_i \left(\sum_m p_i^m - 1\right) - \sum_{i,m,r} \gamma_{i,r}^m (\psi_i^m - \psi_{i,r}(p_i^m))$$

with $a, \gamma \geq 0$.

With the complementary slackness condition and stationary conditions we get:

$$\begin{aligned} \lambda_i^m p_i^m &= 0 \qquad \forall i, m \\ \frac{\partial \mathcal{L}}{\partial p_i^m} &= -u_i^m - a_i^m + b_i + \sum_r \gamma_{i,r}^m (1 + \log(p_{i,r})) \qquad \forall i, m \\ \gamma_{i,r}^m (\psi_i^m - \psi_{i,r}(p_i^m)) &= 0 \qquad \forall i, m, r \\ \frac{\partial \mathcal{L}}{\partial \psi_i^m} &= 1 - \sum_r \gamma_{i,r}^m \end{aligned}$$

Meaning that for any feasible condition we get $\gamma_{i,r}^m = 0$ if $r \notin argmax\{\psi_{i,r}(p_i^m)\}$ and $a = 0$. Then, $\left|\sum_r \gamma_{i,r}^m \log(p_{i,r}) - \log(p_i^m)\right| \leq \varepsilon_R$, meaning that:

$$\begin{aligned} u_i^m - b_i - 1 - \varepsilon_R \leq \log(p_i^m) &\leq u_i^m - b_i - 1 + \varepsilon_R \\ \frac{e^{u_i^m}}{e^{1+b_i}} \cdot e^{-\varepsilon_R} \leq p_i^m &\leq \frac{e^{u_i^m}}{e^{1+b_i}} \cdot e^{\varepsilon_R} \end{aligned}$$

with $\sum_m e^{u_i^m} \cdot e^{-\varepsilon_R} \leq e^{1+b_i} \leq \sum_m e^{u_i^m} \cdot e^{\varepsilon_R}$, so:

$$\frac{e^{u_i^m}}{\sum_{m'} e^{u_i^{m'}}} \cdot e^{-2\varepsilon_R} \leq p_i^m \leq \frac{e^{u_i^m}}{\sum_{m'} e^{u_i^{m'}}} \cdot e^{2\varepsilon_R}$$